STUDY GUIDE:

Module 12: Arithmetic As The Gateway To Algebra

In this module we further explore a topic that has already been introduced in earlier modules. Namely, arithmetic and algebra both arise from a common source--*formulas*.

For example, if we take the formula that relates the marked price (M) to the total price (T) if there is a 5% sales tax:

$$1.05 \times M = T \qquad (1)$$

we get an arithmetical equation if we're given M and want to find the value of T. But if we're given the value of T and want to find the value of M, then we have an algebraic equation.

By way of illustration suppose we know that the marked price is $30. We replace M by 30 in (1) to get:

$$1.05 \times 30 = T \qquad (2)$$

and to solve (2) we need only do the indicated arithmetic.

However, suppose we're told that the total price is $63. This time it's T that we replace by 63 in (1) to get:

$$1.05 \times M = 63 \qquad (3)$$

Because the "unknown" is being used as part of the arithmetic in (3), we call (3) an algebraic equation. In summary, a formula is neither arithmetic nor algebra; but depending on what "blank" we have to fill in, we get either an arithmetic problem or an algebra problem.

Step 1:

View Videotape Lecture # 12.

Step 2:

(i) Read Module 12 of the textbook.

(ii) View Videotape Lecture 12B

Step 3:

When you feel you understand the material presented in Steps 1 and 2, complete the following "Check-The-Main-Ideas" self-quiz by correctly filling in each blank.

Check the Main Ideas:

A relationship such as T + 7 = R is called a _____. The formula becomes an arithmetical equation if we're given the value of _____ and have to find the value of _____. It becomes an algebraic equation if we're given the value of _____ and have to find the value of _____.

formula
T
R
R
T

Sometimes it is psychologically easier if we can translate a formula into words. Thus T + 7 = R means that we start with T, then _____ 7, and the answer is _____. We don't know what the value of R will be until we know the value of T. For example, if T is 9, T + 7 = _____, hence R is _____. But if T = 12, T + 7 = _____, hence R is now _____.

add
R
16; 16
19; 19

Now suppose we were given the value of R. For example if R is 12, T + 7 = R becomes the algebraic equation:

_____ + 7 = _____ (1)

T; 12

To solve equation (1) we have to subtract _____ from both sides of the equation to get:

7

_____ = _____

T; 5

If we want to write m = 3n verbally, we say:

1. Start with ____. *n*
2. _____ by 3. *multiply*
3. The answer is _____. *m*

So if we're given m, we simply reverse the steps, replacing each arithmetic operation by its inverse. For example, if we know that m = 21 we find n by _____ by 3. The fact that there was no arithmetical symbol between 3 and n meant that we _____ 3 and n. *dividing* *multiplied*

If the formula is s = 3(t + 5), it means that we start with _____, then ____ 5, then _____ by 3. The answer will then be ____. That is: *t; add; multiply* *s*

1. Pick t
2. Add 5
3. Multiply by 3
4. The answer is s

So if t = 7, s = _____. *36*

If we wanted to find the value of t given the value of s, the steps would become:

1. Pick s
2. _____ by _____ *Divide; 3*
3. _____ 5 *Subtract*
4. The answer is t.

For example if s = 27, t = _____. *4*

Grouping is important. For example if we write the formula as s = (3t) + 5, we start with t and _____ by 3. Then we add _____. *multiply; 5*

That is:

1. Pick t
2. Multiply by 3
3. Add 5
4. The answer is s.

So if t = 7, s = _____. *26*

The point is that we do what's inside the _____ first. *parentheses*

Once we understand what the equation means it is often easier to solve it without using all the extra words. For example, if we want to find the value of c for which c X 6 = 84, we simply ____ both sides by _____ and get that c = ____. *divide; 6; 14*

As more operations are introduced the equations look more complicated, but how we approach them always remains the same.

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Step 4:

Do the Mastery Review.

Mastery Review:

Answers:

1. Fill in the blank: _____ X 7 = 280. 1. _____
2. Rewrite ___ ÷ 5 = 30 as a fill-in-the-blank problem that can be done directly with a hand calculator. 2. _____
3. Solve for n if 12 X 8 = n. 3. _____
4. Solve for t if 23 + 11 = t. 4. _____
5. Solve for q if 27 + q = 55. 5. _____

(cont)

Mastery Review: (cont)

Answers:

6. Which of the following is an algebraic equation:
   (a) 12 X 3 = 36
   (b) 12 X 3 = m
   (c) 12 X m = 36
   (d) m = 12 X 3

   6. _____

7. Given the formula M X 1.05 = T, solve for T if M = 50.

   7. _____

8. Given the formula M X 1.05 = T, solve for M if T = 105.

   8. _____

9. The variables b and c are related by the formula b = c + 9. Find the value of c when b = 15.

   9. _____

10. Use the undoing method to solve the equation c - 15 = 14.

    10. _____

11. Use the undoing method to solve the equation c X 6 = 48.

    11. _____

12. In the formula W = 7P, find the value of W when P = 6.

    12. _____

13. In the formula W = 7P, find the value of P when W = 70.

    13. _____

14. Translate X = Y - 4 into a verbal formula.

    14. _____

15. Invert the formula:

    Step 1: Start with q

    Step 2: Divide by 5

    Step 3: The answer is u

    15. _____

16. Use the formula below to determine the value of C when P = 7.

    (1) Start with P
    (2) Multiply by 5
    (3) Add 6
    (4) The answer is C.

    16. _____

17. Use the formula below to find the value of P when C = 46.

    (1) Start with P
    (2) Multiply by 5
    (3) Add 6
    (4) The answer is C.

    17. _____

(cont)

Mastery Review (cont)

Answers:

18. Use the formula:
$$C = \frac{5}{9}(F - 32)$$
to find the Celsius temperature when the Fahrenheit temperature is 77°.

18. _____

19. Convert $70^{\circ}F$ into an equivalent Celsius temperature.

19. _____

20. What is the Fahrenheit reading if the Celsius reading is 40?

20. _____

21. Use the formula $d = 16(t^2)$ to find the value of d if $t = 5$.

21. _____

22. Use the formula $d = 16(t^2)$ to find out how far an object falls in 1/2 minute.

22. _____

23. Given the formula $d = 16(t^2)$ find the value of t when $d = 1{,}296$.

23. _____

Answers:

1. *40*
2. *__ = 5 X 30*
3. *n = 96*
4. *t = 34*
5. *q = 28*
6. *(c)*
7. *T = 52.5*
8. *M = 100*
9. *c = 6*
10. *c = 29*
11. *c = 8*
12. *W = 42*
13. *P = 10*
14. *Start with Y*
    Subtract 4
    The answer is X
15. *Start with u*
    Multiply by 5
    The answer is q.
16. *C = 41*
17. *C = 8*
18. *$25^{\circ}C$*
19. *$21\frac{1}{9}\ ^{\circ}C$*
20. *$104^{\circ}F$*
21. *d = 400*
22. *14,400 feet*
23. *t = 9*

Step 5:

Do Self-Test 12, Form A on the next page.

Self-Test 12, Form A

ANSWERS:

1. For what value of n will:

   (a) $n = 19.24598 + 52.1347$?

   (b) $n + 19.24598 = 52.1347$?

   1. (a) _____

      (b) _____

2. Rounded off to the nearest hundredth, for what value of q will:

   $168.43 = 7.19(q + 18.97)$?

   2. _____

3. Rounded off to the nearest hundredth, for what value of q will:

   $168.43 = 7.19q + 18.97$?

   3. _____

4. For what value of t will

   (a) $16t^2 = 1{,}024$?

   (b) $(16t)^2 = 1{,}024$?

   4. (a) _____

      (b) _____

5. (a) Evaluate $(t^2 + 5)t$ when $t = 2$.

   (b) Evaluate $(t^2 + 5)t$ when $t = 3$.

   (c) Evaluate $(t^2 + 5)t$ when $t = 2.5$?

   (d) Rounded off to the nearest whole number, find a value of t for which:

   $$(t^2 + 5)t = 29$$

   HINT: Use parts (a), (b), and (c) to do part (d).

   5. (a) _____

      (b) _____

      (c) _____

      (d) _____

6. If $V = LWH$, find the value of H if $V = 240$, $L = 6$, and $W = 5$.

   6. _____

7. If $P = 2(L + W)$, find the value of L if $P = 320$ and $W = 70$.

   7. _____

8. If $S = \frac{n}{2}(a + L)$, find the value of n if $S = 300$ when $a = 23$ and $L = 17$.

   8. _____

9. An object falls h feet in t seconds according to the rule $h = 16t^2$. To the nearest 10th of a second, how long will it take for it to fall 700 feet?

   9. _____

10. The area (A) of a circle is related to its radius (r) by the formula: $A = \pi r^2$. To the nearest tenth of an inch, find the radius of a circle whose area is 863 square inches. Let 3.1416 stand for π.

    10. _____

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 12, Form A

1. *(a) 71.38068 (b) 32.88872*
2. *4.46*
3. *20.79*
4. *(a) 8 (b) 2*
5. *(a) 18 (b) 42 (c) 28.125 (d) 3*
6. *8*
7. *90*
8. *15*
9. *6.6 seconds*
10. *16.6 inches*

****

If you did each problem in Self-Test 12, Form A correctly, you may, if you wish, proceed to *The Final Step* at the end of this study guide module. Otherwise, continue with Step 6.

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Step 6:

Study the solutions for Self-Test 12, Form A with special emphasis on any problems you failed to answer correctly.

Solutions for Self-Test 12, Form A

1.

This exercise reinforces our remarks about reading comprehension. Notice how much parts (a) and (b) resemble one another; yet they are very different.

(a)

The right hand side of

$$n = 19.24598 + 52.1347 \qquad (1)$$

simply tells us to add the two given constants. That is:

$$\begin{array}{r} 19.24598 \\ +\ 52.1347 \\ \hline 71.38068 \end{array}$$

Here again we see the value of the calculator. Enter 19.24598, press the "+" key, enter 52.1347, press the "=" key.

Thus we may rewrite (1) as:

$$n = 71.38068$$

In terms of the discussion in this module, (1) is an arithmetical equation and can be solved as is just by using a calculator.

When we say "using a calculator" we mean just do the actual arithmetic.

(b)

In the equation

$$n + 19.24598 = 52.1347 \qquad (2)$$

n is part of the arithmetic. That is, (2) is an algebraic equation. One way to solve it is to "isolate" n. Since 19.24598 is being added to n, we undo it by subtracting 19.24598 from both sides of (2) to get:

The key difference between parts (a) and (b) is that in (a) the variable is by itself on one side of the equation. In (b) the variable is part of the addition.

Remember that to maintain an equality whatever we do on one side of the equation, we also have to do on the other.

1. (b) (cont)

$$\begin{array}{rcl} n + 19.24598 & = & 52.1347 \\ -\ 19.24598 & & -\ 19.24598 \\ \hline n & = & 32.88872 \end{array}$$

Using the calculator: Enter 52.1347, press the "-" key, enter 19.24598, press the "=" key. But the calculator won't tell you that you have to subtract!

If you feel more comfortable with the verbal form, rewrite the problem as:

1. Start with n → → → → → → *The answer is n* (3)
2. Add 19.24598 → → → → → → *Subtract 19.24598* (2)
3. The answer is 52.1347 → → *Start with 52.1347* (1)

The italics describe the "inverse" formula that we need here. Notice that it tells us the same thing that the undoing method told us.

Whatever method we use, we can always check whether we've done the problem correctly. Namely if we replace n by 32.88872 in (2) we should get a true statement. In fact:

$$\begin{array}{r} n \\ \downarrow \\ 32.8872 \end{array} + 19.24598 = 52.1347$$

$$\begin{array}{r} 32.88872 \\ +\ 19.24598 \\ \hline 52.13470 \end{array} \text{ or } 52.1347$$

2.

Exercises 2 and 3 show us the importance of grouping symbols (parentheses) as well as the value of having a calculator.

We're working with the algebraic equation:

$$168.43 = 7.19(q + 18.97) \qquad (3)$$

We have to remember that we do what's in the parentheses first. The variable is q. So in verbal form the equation says:

That is, whatever is within the parentheses is treated as one number.

(1) Start with q
(2) Add 18.97
(3) Multiply by 7.19
(4) The answer is 168.43

Don't multiply by 7.19 first. q and 18.97 are within the parentheses, so we have to add first.

(cont)

2. (cont)

Inverting, we get:

(1) Start with 168.43

(2) Divide by 7.19

(3) Subtract 18.97

(4) The answer is q

On the calculator, enter 168.43, press the "÷" key, enter 7.19, press the "=" key, press the "-" key, enter 18.97, press the "=" key. The displayed answer is 4.4555911 which rounds off to 4.46

That is:

(1) Start with q → → → → → The answer is q
(2) Add 18.97 → → → → → → Subtract 18.97
(3) Multiply by 7.19 → → → Divide by 7.19
(4) The answer is 168.43 → → Start with 168.43

Don't look for shortcuts. You have to retrace your steps exactly or you'll get the wrong answer.

If you prefer to use the undoing method without first translating everything into words, notice that to "unblock" the parentheses we have to "get rid of" 7.19. Since 7.19 is multiplying the parentheses, we divide both sides by 7.19. That is:

$$\frac{168.43}{7.19} = \frac{\cancel{7.19}(q + 18.97)}{\cancel{7.19}}$$

168.43/7.19 means that we are dividing 168.43 by 7.19

or

$$23.425591 = q + 18.97$$

and to undo adding 18.97 to q we subtract it from both sides to get:

$$\begin{array}{rcl} 23.425591 & = & q + \cancel{18.97} \\ -\ 18.97 & & -\ \cancel{18.97} \\ \hline 4.455591 & = & q \end{array}$$

The exact answer, of course, will depend on how accurately we rounded off at each arithmetic step.

It is crucial that you first divide by 7.19 and then subtract 18.97. The reason is that in the expression 7.19(q + 18.97) both q and 18.97 are being multiplied by 7.19. That is, we could

(cont)

2. (concluded)

apply the distributive property to convert

$$168.43 = 7.19(q + 18.97)$$

into

$$168.43 = 7.19q + 7.19(18.97)$$

or

$$168.43 = 7.19q + 136.3943$$

I simply multiplied 7.19 by 18.97 on the calculator to get 136.3943

In words:

(1) Start with q

(2) Multiply by 7.19

(3) Add 136.3943

(4) The answer is 168.43

This is another role of algebra. The "programs"

Start with q
Multiply by 7.19
Add 136.3943
The answer is 168.43

and

Start with q
Add 18.97
Multiply by 7.19
The answer is 168.43

don't look alike; yet they are equivalent.

Now if we reverse the steps, we see that we first subtract 136.3943 and then divide by 7.19.

This idea is discussed further in our treatment of Exercise 3. For the time being, however, remember that to check your answer--no matter how you did the problem--you replace q by 4.455591 in (3) and make sure that you get a true statement.

If you replace q by 4.46, you'll get an "almost-true" statement. That is, whenever we round off, we lose some accuracy.

3.

Recall that 7.19q + 18.97 is assumed to be grouped as (7.19q) + 18.97. *Except for the placement of the parentheses, Exercises 3 and 2 look identical. But the grouping makes a difference.* Namely this time we first multiply by 7.19 and then add 18.97. That is:

$$168.43 = 7.19q + 18.97 \qquad (4)$$

This is an accepted shortcut. But if you feel more comfortable, always use the grouping symbols.

says in words:

(1) Start with q

(2) Multiply by 7.19

(3) Add 18.97

(4) The answer is 168.43

If we invert the "program" we get:

(1) Start with q → → → → → → The answer is q
↓ ↑
(2) Multiply by 7.19 → → → → → Divide by 7.19
↓ ↑
(3) Add 18.97 → → → → → → → → Subtract 18.97
↓ ↑
(4) The answer is 168.43 → → → Start with 168.43

If we want to use the undoing method directly, we start with

$$168.43 = (7.19q) + 18.97 \qquad (4)$$

Since everything within the parentheses is one number, we first have to isolate the parentheses. To do this we note that 18.97 is being added to the parentheses. So to undo this we subtract 18.97 from both sides to get:

$$\begin{array}{rl} & 168.43 = 7.19q + 18.97 \\ - & \underline{18.97 = \qquad\quad - 18.97} \\ & 149.46 = 7.19q \end{array} \qquad (5)$$

Since 7.19 is multiplying q, we undo this by dividing both sides of (5) by 7.19 to get:

$$\frac{149.46}{7.19} = \frac{\cancel{7.19}q}{\cancel{7.19}}$$

or

$$20.787204 = q \qquad (6)$$

which to the nearest hundredth says that q = 20.79.

If we compare this with Exercise 2, we see that "all" we did was interchange (2) and (3). While this looks "natural" it's still a big difference. Namely if we add 18.97 first and then multiply by 7.19, the 18.97 is also multiplied by 7.19. But if we add it afterwards, it isn't multiplied by 7.19.

The inversion doesn't depend on using 168.43. If we let A denote the "answer", we have:

$$(A - 18.97) \div 7.19 = q$$

Using the calculator, we'd enter 168.43, press the "-" key, enter 18.97, press the "=" key, press the "÷" key, enter 7.19, press the "=" key.

Because we used the rules of arithmetic to get from (4) to (6), equations (4), (5) and (6) are simply three different forms of the same relationship. In essence, another phase of algebra is that it is the process that gets from (4) to the simpler equation (6).

4.

This exercise continues the theme of the previous ones, but emphasizes the inverse relationship that exists between the "square" and the "square root"

(a)

We have the algebraic equation:

$$16t^2 = 1{,}024 \qquad (1)$$

which is read the same way as is:

$$16(t^2) = 1{,}024$$

Since t is within the parentheses, we have to "clear" the parentheses first. We observe that 16 is multiplying the parentheses. So to undo this, we diivde both sides of the equation by 16 to get:

$$\frac{16t^2}{16} = \frac{1{,}024}{16}$$

or

$$t^2 = 64 \qquad (2)$$

In (2), t is being squared, so to undo this we take the square root of both sides to get:

$$\sqrt{t^2} = \sqrt{64}$$

or

$$t = 8 \qquad (3)$$

It's like a tongue-twister. The square root of the square of a number is the number itself.

In words, we have:

(1) Start with t → → → → → The answer is t
(2) Square it → → → → → → → → Take the square root
(3) Multiply by 16 → → → → Divide by 16
(4) The answer is 1,024 → → → Start with 1,024

Using the calculator, enter 1,024, press the "÷" key, enter 16, press the "=" key, press the "$\sqrt{x}$" key. 8 then appears as the answer.

4. (b)

This time the square is outside the parentheses. This means that we square after we multiply t by 16. That is, in words

$$(16t)^2 = 1{,}024 \qquad (4)$$

says:

(1) Start with t

(2) Multiply by 16

(3) Square

(4) The answer is 1,024

So to invert in this case, we'd start with 1,024, then take the square root (32), and then we divide by 16 (2) and we see that t = 2.

In terms of undoing (4), we notice that the right side of the equation is the square of what's in the parentheses, so we first take the square root of both sides to get:

$$16t = 32$$

from which it easily follows that t = 2.

Note that as usual there is more than one way to do a problem. For example, if you wish you may rewrite $(16t)^2$ *as (16t)(16t) or (16)(16)(t)(t) or* $256t^2$. *In this way, (4) becomes:*

$$256t^2 = 1{,}024 \qquad (5)$$

We can then divide both sides by 256 to get $t^2 = 4$ *and we can then take the square root of both sides to get t = 2.*

Check

Start with 2

Multiply by 16 (32)

square 32 (1,024)

Depending on where the parentheses areplaced, we have either:

Start with t
Square it
Multiply by 16
The answer is 1,024

or

Start with t
Multiply by 16
Square it
The answer is 1,024

The first form tells us that t = 8, the second tells us that t = 2. So the order of the steps--as indicated by the parentheses--is very important.

Notice that in form (1) and (5) are the same.

5.

In a sense, this is a "sneaky" problem. It's designed to show you that algebra may be simple in concept but often difficult to apply. Moreover, when the algebra is difficult to apply, we can often get an approximate answer if we understand arithmetic.

To see what we're driving at here, let's look at part (d) first. The algebraic equation:

$$(t^2 + 5)t = 29 \qquad (1)$$

is not difficult to put into words. Namely, if we start within the parentheses, we have:

(1) Start with t

(2) Square it

(3) Add 5

(4) Multiply by t

(5) The answer is 29

We'll use steps (1) through (4) --with t = 2, t = 3, and t = 2.5--in doing parts (a), (b), and (c).

The problem occurs after we invert the relationship. Namely:

(1) Start with t → → → → → The answer is t.
↓ ↑
(2) Square it → → → → → → Take the square root
↓ ↑
(3) Add 5 → → → → → → → → Subtract 5
↓ ↑
(4) Multiply by t → → → → Divide by t
↓ ↑
(5) The answer is 29 — — Start with 29

Here's where the problem lies. We don't know what the value of t is, so how can we divide by it? That is, to use the undoing method we have to be dealing with constants, not variables.

We'd have to know the value of t in order to divide by t; but if we knew the value of t we wouldn't be doing this problem.

5. (cont)

The point is that to do part (d), we'd first do parts (a), (b), and (c). What we'll be doing is "pinning" the value of t between two consecutive whole numbers by finding values of t that give as one answer that's too small and another that's too big.

(a)

We start with $(t^2 + 5)t$ and replace t by 2 That is:

That is, we pick 2, square it, add 5, multiply by 2, and that's the answer.

$$(t^2 + 5)t$$
$$\downarrow \quad \downarrow$$
$$(2^2 + 5)2 =$$
$$(4 + 5)2 =$$
$$(9)2 =$$
$$18$$

Remember that (9)2 is another way of saying 9 X 2.

This tells us that t = 2 cannot be a solution of the equation $(t^2 + 5)t = 29$, because 18 is too small.

(b)

Again we start with $(t^2 + 5)t$ but this time we replace t by 3. That is:

$$(t^2 + 5)t$$
$$\downarrow \quad \downarrow$$
$$(3^2 + 5)3 =$$
$$(9 + 5)3 =$$
$$(14)3 =$$
$$42$$

This tells us that t = 3 cannot be a solution of the equation $(t^2 + 5)t = 29$, because 42 is too big

Parts (a) and (b) tell us that the value of t we're seeking must be greater than 2 but less than 3. Part (c) will tell us which number t is closer to.

This idea will explored in more detail in our solution of Exercise 9.

5 (c)

Again we start with $(t^2 + 5)t$ but this time we replace t by 2.5 to get

$$(2.5^2 + 5)2.5 =$$

$$(6.25 + 5)2.5 =$$

$$(11.25)2.5 =$$

$$28.125$$

Hence $t = 2.5$ can't be a solution of $(t^2 + 5)t = 29$ because 28.125 is too small.

(d)

We are now in an excellent position to use the results of parts (a), (b), and (c) to do (d). Look at the results of these 3 parts:

$t = 2$: $(t^2 + 5)t = 18$

$t = 2.5$: $(t^2 + 5)t = 28.125$

.....

$t = 3$: $(t^2 + 5)t = 42$

29

See why we're using 2.5? Any number between 2 and 2.5 is closer to 2 than to 3; and any number between 2.5 and 3 is closer to 3 than to 2.

Since 29 is between 28.125 and 42, the value of t must be between 2.5 and 3. Since any number between 2.5 and 3 is closer to 3 than to 2, we see that to the nearest whole number the value of t must be 3.

Notice that in (d) we were not asked for the exact value of t (which would be an advanced algebra problem). All we needed was an answer rounded off to the closest whole number.

The idea of solving algebraic equations by taking guesses which give us over-estimates and under-estimates is a very important part of the practical course called NUMERICAL ANALYSIS. Especially with calculators and computers this technique gives us excellent estimates very quickly.

This is another reason why it's very important to UNDERSTAND arithmetic before studying algebra or, for that matter, any other mathematics course.

6.

Up to now we've simply begun with an algebraic equation. Starting with this exercise, we try to show where these equations come from. For the most part algebraic equations come from formulas that have two or more variables. If we're given the value of all but one variable, we can get an algebraic equation that will have only one variable. For example, in the formula:

$$V = LWH \qquad (1)$$

we have four variables; V, L, W, and H.

V = LWH happens to be the formula that tells us how to find the volume (V) of a rectangular box whose length is L, whose width is W and whose height is H. But we don't have to know this to do Exercise 6!

We're told that V = 240, L = 6, and W = 5; and we're asked to find the value of H.

Rewriting (1) as

$$V = L \times W \times H$$

we have:

We're using the "times sign" to emphasize that this is a multiplication problem.

$$\begin{array}{l} V = L \times W \times H \\ \downarrow \quad \downarrow \quad \downarrow \\ 240 = 6 \times 5 \times H \end{array}$$

$$240 = (6 \times 5) \times H$$

$$240 = 30 \times H$$

$$240 \div 30 = H$$

or

$$H = 8$$

As soon as we replaced V, L, and W by their given values we got an algebraic equation with H as the only variable.

Once we've got the algebraic equation, we proceed just as we did in the earlier exercises.

Check: $240 = 6 \times 5 \times 8$ *is a true statement*

7.

You may remember this formula as the one we used in finding the perimeter (P) of a rectangle whose length was L and whose width was W. But this isn't important in this exercise. What is important is that we shall replace P and W by their given values to get an algebraic equation in which L will be the variable (unknown). We'll then solve this equation for the value of L.

Hence:

$$\begin{array}{ccccc} P & = & 2(L & + & W) \\ \downarrow & & & & \downarrow \\ 320 & = & 2(L & + & 70) \end{array} \qquad (1)$$

If you want to think of (1) in terms of perimeter, we're being asked to find the length of a rectangle if its width is 70 (feet) and its perimeter is 320 (feet). Remember that to use this formula for perimeter; P, L, and W have to be in a common unit.

To solve (1) we first divide both sides by 2 to get:

$$160 = L + 70$$

and we then subtract 70 from both sides to get:

$$\begin{array}{rl} & 160 = L + 70 \\ - & \underline{70 \quad\quad - 70} \\ & 90 = L \end{array}$$

We omit some of the details because they're the same as in our earlier exercises.

In words:

Start with L (90)
Add 70 (160)
Multiply by 2 (320)
The answer is 320.

To undo this, start with 320, divide by 2, and add 70 to get 90 as the answer.

8.

This formula is used for computing a very special case of sum. It will be explained more in Videotape Lecture 12S, but again we don't have to know what the formula means in order to be able to use it.

What we'll do here is replace S by 300, a by 23, and L by 17 to get:

$$300 = \frac{n}{2}(23 + 17)$$

or

$$300 = \frac{n}{2}(40)$$

$$= \frac{n}{2} \times 40$$

$$= \frac{n \times 40}{2 \times 1}$$

$$= \frac{40n}{2}$$

or

$$300 = 20n$$

We then divide both sides by 20 to get

$$15 = n$$

It is important to understand that this is how the average professional uses mathematics. The formula is available as is the given information. When this information is correctly put in place the desired equation arises. You don't have to know where the formula comes from to be able to solve the equation.

Remember to work within the parentheses first.

If you don't like fractions, multiply both sides of

$$300 = \frac{n}{2}(23 + 17)$$

by 2 at the very start to get

$$600 = n(40)$$

The important thing is to see that the answer is n = 15, no matter which method you use.

In other words, in the real-world we aren't usually given algebraic equations to solve. Rather they arise after we've analyzed a given situation and replaced the variables in a formula by numbers. This is illustrated in more detail in the last two exercises.

9.

All that makes this formula different from the ones in Exercises 6, 7, and 8 is that now we're told what the formula applies to. But rather than being told that h = 700, we have to decide this for ourselves after we've read the problem.

Once we've done this the problem is done exactly as the others. Namely, we replace h by 700 and solve for t. That is:

$$\begin{array}{l} h = 16(t^2) \\ \downarrow \\ 700 = 16(t^2) \end{array} \qquad (1)$$

The parentheses are supplied just for emphasis.

In words, we have:

Start with t → → → → → The answer is t.
↓ ↑
Square it → → → → → → → Take the square root
↓ ↑
Multiply by 16 → → → Divide by 16
↓ ↑
The answer is 700 → → → Start with 700

This problem is almost identical with those we did as Examples in the Module. The only difference is that before we chose examples in which the height was divisible by 16 and all square roots were whole numbers.

So to solve this exercise we have:

1. Start with 700 (feet)
2. Divide by 16 *(43.75)*
3. Take the square root *(6.6143783)*
4. The answer is the time in seconds. *(6.6143783 seconds)*

See the advantage of having a square root key on your calculator?

Since we want the time rounded off to the nearest tenth of a second, our answer is:

t = 6.6 seconds

(cont)

9. (cont)

A Note Concerning Exercise 5:

Suppose you didn't have a calculator. You could still have computed heights for different values of time. For example:

If t = 1, h is 16(1^2) or 16 feet

If t = 2, h is 16(2^2) or 64 feet

If t = 3, h is 16(3^2) or 144 feet

If t = 4, h is 16(4^2) or 256 feet

If t = 5, h is 16(5^2) or 400 feet

If t = 6, h is 16(6^2) or 576 feet } *700 feet*

If t = 7, h is 16(7^2) or 784 feet }

From this chart we see that in 6 seconds the object falls 576 feet which isn't far enough; while in 7 seconds it falls 784 feet, which is too far.

Hence the object has fallen more than 6 seconds but less than 7 seconds by the time it falls 700 feet.

If we wanted further accuracy, we could compute h when t = 6.1, 6.2, 6.3, and so on.

So while algebra is helpful, if we understand the concepts that are involved and if we have convenient computational devices (such as a calculator) we can get as close an estimate as we wish to the right answer simply by making use of our knowledge of arithmetic.

10.

Our concluding exercise is designed to show you how the same type of formula can be used in completely different physical situations. The study of a circle seems to be quite different from the study of a freely-falling body. Yet the formula that related A to r is very much similar to the one that relates h to t.

10. (cont)

In words, the formula:

$$A = \pi r^2 \qquad (1)$$

says:

Start with r

Square it

Multiply by π

The answer is A

Whether we multiply by π or by 16, we're still multiplying by a constant. The fact that π is an irrational number doesn't change the fact that it's a constant.

The inversion says:

Start with A

Divide by π

Take the square root

The answer is r

Since we're told that A is 863 (square inches) and that we're to use 3.1416 for π, all we have to do is:

Start with 863 (square inches)

Divide by 3.1416 (to get 274.70079)

Take the square root (to get 16.5741)

The answer is r (in square inches)

It's no hardship not to round off yet because all these digits are "locked" into my calculator and all I have to do next is press my square root key. Without the calculator, the computation is at the least very tedious.

So to the nearest tenth of an inch, the radius of the circle is 16.6 inches.

As a check we can compute the area of a circle that has a radius of 16.5 inches, 16.55 inches, and 16.6 inches, which is exactly the strategy we used in Exercise 5. Any way, we get:

Solutions for Self-Test 12, Form A (concluded)

10. (concluded)

If r = 16.5: $A = 3.1416(16.5^2)$

$= 3.1416(272.25)$

$= 855.3006$ *(less than 863)*

If r = 16.55: $A = 3.1416(16.55^2)$

$= 3.1416(273.9025)$

$= 860.49209$ *(still less than 863)*

If r = 16.6: $A = 3.1416(16.6^2)$

$= 3.1416(275.56)$

$= 865.6993$ *(greater than 863)*

So the radius must be between 16.55 and 16.6 inches. To the nearest tenth of an inch, this rounds off to 16.6 inches.

Hopefully, you are now beginning to see how closely arithmetic and algebra are related. It is important that you understand that once you understand the formula and know arithmetic, you can solve most equations approximately even if algebra gives you difficulty.

****

Step 7:

Do Self-Test 12, Form A on the next page.

Self-Test 12, Form B

1. For what value of n will:

   (a) $n = 14.98376 + 38.2367$?

   (b) $n + 14.98376 + 38.2367$?

2. Rounded off to the nearest hundredth, for what value of q will:

   $193.92 = 8.23(q + 19.67)$?

3. Rounded off to the nearest hundredth, for what value of q will:

   $193.92 = 8.23q + 19.67$?

4. For what value of t will

   (a) $4t^2 = 1{,}600$?

   (b) $(4t)^2 = 1{,}600$?

5. (a) Evaluate $(t^2 + 3)t$ when $t = 4$.

   (b) Evaluate $(t^2 + 3)t$ when $t = 5$.

   (c) Evaluate $(t^2 + 3)t$ when $t = 4.5$

   (d) Rounded off to the nearest whole number, find a value of t for which:

   $(t^2 + 3)t = 106$

6. If $V = LWH$, find the value of H if $V = 350$, $L = 10$ and $W = 7$.

7. If $P = 2(L + W)$, find the value of L if $P = 280$ and $W = 65$.

8. If $S = \frac{n}{2}(a + L)$, find the value of n if $S = 250$ when $a = 21$ and $L = 29$.

9. An object falls h feet in t seconds according to the rule $h = 16t^2$. To the nearest tenth of a second, how long will it take for the object to fall 850 feet?

10. The area of a circle (A) is related to its radius (r) by the formula: $A = \pi r^2$. To the nearest tenth of an inch, find the radius of a circle if its area is 725 square inches. Let 3.1416 stand for π.

ANSWERS:

1. (a) _____

   (b) _____

2. _____

3. _____

4. (a) _____

   (b) _____

5. (a) _____

   (b) _____

   (c) _____

   (d) _____

6. _____

7. _____

8. _____

9. _____

10. _____

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 12, Form B

1. *(a) 53.22046 (b) 23.25294*
2. *3.89*
3. *21.17*
4. *(a) 20 (b) 10*
5. *(a) 76 (b) 140 (c) 104.625 (d) t = 5*
6. *H = 5*
7. *L = 75*
8. *n = 10*
9. *7.3 seconds*
10. *15.2 inches*

****

If you did each problem in Self-Test 12, Form B correctly, you may, if you wish, proceed to *The Final Step* at the end of this module in the Study Guide. Otherwise, continue woth Step 8.

****

Step 8:

View the solutions for Self-Test 12, Form B on Videotape Lecture 12S. Pay special attention to the solutions of those problems for which you failed to get the correct answers.

Step 9:

Do Self-Test 12, Form C on the next page.

Self-Test 12, Form C

ANSWERS:

1. For what value of n will:

   (a) $n = 12.98234 + 57.4378$?

   (b) $n + 12.98234 = 57.4378$?

   1. (a) _____

      (b) _____

2. Rounded off to the nearest hundredth, for what value of q will:

   $178.17 = 9.21(q + 12.89)$?

   2. _____

3. Rounded off to the nearest hundredth, for what value of q will:

   $178.17 = 9.21q + 12.89$?

   3. _____

4. For what value of t will

   (a) $9t^2 = 324$?

   (b) $(9t)^2 = 324$?

   4. (a) _____

      (b) _____

5. (a) Evaluate $(t^2 + 4)t$ when $t = 7$.

   (b) Evaluate $(t^2 + 4)t$ when $t = 8$.

   (c) Evaluate $(t^2 + 4)t$ when $t = 7.5$

   (d) Rounded off to the nearest whole number find a value of t for which:

   $(t^2 + 4)t = 453$

   5. (a) _____

      (b) _____

      (c) _____

      (d) _____

6. If $V = LWH$, find the value of H is $V = 420$ when $L = 7$ and $W = 3$.

   6. _____

7. If $P = 2(L + W)$, find the value of L if $P = 300$ and $W = 85$.

   7. _____

8. If $S = \frac{n}{2}(a + L)$, find the value of n if $S = 150$ when $a = 11$ and $b = 19$.

   8. _____

9. An object falls h feet in t seconds according to the rule $h = 16t^2$. To the nearest tenth of a second, how long will it take for the object to fall 950 feet?

   9. _____

10. The area (A) of a circle is related to its radius (r) by the formula: $A = \pi r^2$. To the nearest tenth of an inch, find the radius of a circle if its area is 935 square inches. Let 3.1416 stand for π.

    10. _____

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 12, Form C

1. *(a) 70.42014* *(b) 44.45546*

2. *6.46*

3. *17.95*

4. *(a) 6* *(b) 2*

5. *(a) 371* *(b) 544* *(c) 451.876* *(d) t = 8*

6. *H = 20*

7. *L = 65*

8. *n = 10*

9. *7.7 seconds*

10. *17.3 inches*

****

THIS CONCLUDES OUR STUDY GUIDE PRESENTATION OF MODULE 12.

HOPEFULLY, YOU WILL NOW FEEL READY TO BEGIN *THE FINAL STEP*, WHICH IS PRESENTED ON THE NEXT PAGE.

HOWEVER, IF YOU STILL FEEL THAT YOU NEED MORE TIME AND/OR HELP BEFORE CONTINUING YOU SHOULD CONSULT WITH A TRACHER, A FRIEND, OR A FELLOW-STUDENT FOR ADDITIONAL REINFORCEMENT.

****

Wrapping it up ------

THE FINAL STEP:

Congratulations for getting to this point in our course. For many of you, this was probably no easy achievement, especially if you fell into the "math anxiety" category. Yet, regardless of whether you had self-doubts about taking this course, the fact is that you had to get perfect scores in 12 Tests to get to this point. This, alone, should be enough reason for you to have an increased confidence level in mathematics. Of perhaps even more importance is the fact that success is often "contagious". That is, achieving success in one area often leads to the confidence to succeed in other areas of endeavor. And as you continue to succeed you will find yourself developing in new directions of personal and professional growth.

But growth brings with it the need for further development. Our hope is that this course is just a first step toward developing your mathematical skills and your ability to think critically. In this respect it is important that you *retain* what you've learned in this course. It has been said that *when you've forgotten everything else you ever learned; what's left is the* only *thing you can call* education. Knowledge that is soon forgotten is really no education at all. For this reason we feel it is important that you have a chance to measure what you have retained from the course.

So please take whatever time you need to review the 12 Self-Tests and make sure that you *think* you understand the various skills and techniques that were discussed throughout the course. After you feel you understand what you've learned do the *Final Mastery Review* that follows this discussion. This review consists of 30 questions which we've chosen as being representative of the material we've already been studying in this course. *It should not be thought of as a final examination. It is simply a device to see how much you've*

retained from the course. In short, this review is simply a diagnostic tool that should help you judge your readiness for taking the next mathematics course--be it business math, algebra, or statistics. Following the review, we have presented the answers to all the problems as well as a brief outline of the solution for each problem. If you got any problem wrong, you should check our solution and if there is still doubt in your mind; look for the appropriate portions of the textbook and study guide for further review. Should you still need more drill, do the *Final Mastery Review Revisited.*

We had three main goals in mind when we designed this course. Namely:

(1) To Help Students Improve Their Mathematical Self-Image

This goal has already been achieved just by your having reached this part of the course. The material you have learned represents all the ingredients for the study of further mathematics. To have successfully maneuvered through our 12 modules means that you are prepared to proceed with more mathematics courses. Mathematics should no longer be a stumbling block in your desire to achieve further growth.

(2) To Help Students Improve Their Arithmetic Skills

Sometimes feeling unthreatened is the best step towards success. In this sense we wanted to make sure that you were not "threatened" by the ordinary computational skills involved in arithmetic. We wanted you to become at ease with fractions, decimals, mixed numbers and percents. This goal will have been achieved by us if you can do most or all of the first 20 problems in the Final Mastery Review. These 20 problems measure your arithmetic skills.

(3) To Help Students Learn to Apply Arithmetic to Problem-Solving

This is the most difficult goal to reach. Some people never feel comfortable enough with arithmetic to apply it to problem-solving. However, if you fell into this category, you would not have reached this point of our course--since Modules 9, 10, 11, and 12 were devoted almost exclusively to problem-solving in various areas of application. All we can really do is expose you to various techniques and ideas and then hope that you have the aptitude to solve problems on your own. The last 10 problems (there are 30 problems altogether) of the Final Mastery Review are devoted to problem-solving.

But let's face it. In the final analysis you alone are the judge about how you feel about your mathematical skills. At the very worst, by getting this far in our course, you have improved your mathematical skills. At the very best, you are now much more comfortable with mathematical concepts and better able to use your skills toward further avenues of growth. We sincerely hope that this course starts you on a future path that is filled with much professional success and even greater personal happiness.

****

DIRECTIONS FOR TAKING THE FINAL MASTERY REVIEW:

1. Do not look at the Final Mastery Review problems until you are ready to take the Review.

   (This creates the atmosphere for measuring mastery. If you know the questions in advance it is possible that you directed your studying just to these specific questions. The true test of mastery is how well you do with problems that you meet randomly.)

2. Once you've reviewed and start to do the Final Mastery Review, do not make reference to any books or notes

   (In real-life you obviously can use books, notes, and the knowledge of other people. But we want to measure your confidence to be on your own without having books and notes as a "crutch")

3. However, you may use -- in fact, you should use--a calculator in doing the Final Mastery Review.

FINAL MASTERY REVIEW:

ANSWERS:

1. Write forty five trillion, two hundred thirty six billion, eighty nine thousand, twenty seven as a place value numeral.

1. _____

2. What number must we add to 8,389 to get 14,217 as the sum?

2. _____

3. What number must we multiply by 64 to get 19,328 as the product?

3. _____

4. What is the remainder when 39,580 is divided by 704?

4. _____

5. (a) Write 540 as a product of prime numbers.

5. (a) _____

(b) Write 900 as a product of prime numbers.

(b) _____

(c) Write $\frac{540}{900}$ as a common fraction in lowest terms.

(c) _____

6. (a) Write $\frac{3}{8}$ as an equivalent common fraction whose denominator is 40.

6. (a) _____

(b) Write $\frac{3}{8} + \frac{2}{5}$ as a common fraction.

(b) _____

7. (a) What must we add to $\frac{7}{15}$ to get $\frac{2}{3}$?

7. (a) _____

(b) Which is greater and by how much:

$\frac{2}{3}$ or $\frac{7}{15}$?

(b) _____

8. (a) Write $\frac{3}{8} \times \frac{5}{7}$ as a common fraction.

8. (a) _____

(b) How much is $\frac{3}{8}$ of $\frac{5}{7}$?

(b) _____

(c) Is $\frac{3}{8} \times \frac{5}{7}$ less than $\frac{5}{7}$?

(c) _____

9. (a) Write $\frac{3}{4} \div \frac{5}{7}$ as a common fraction.

9. (a) _____

(b) By what must we multiply $\frac{5}{7}$ to get $\frac{3}{4}$?

(b) _____

(c) By what must we multiply $\frac{3}{4}$ to get $\frac{5}{7}$?

(c) _____

(cont)

FINAL MASTERY REVIEW (cont)

ANSWERS:

10. (a) Write $\frac{234}{7}$ as a mixed number. — 10. (a) _____

(b) Write $32\frac{4}{7}$ as a common fraction. — (b) _____

11. Write $7\frac{3}{5} \times 2\frac{5}{6}$ as a mixed number. — 11. _____

12. Write each of the following as a common fraction in lowest terms: — 12. (a) _____

(a) 2.4 — (b) _____

(b) 2.40 — (c) _____

(c) 2.04

13. What must we add to 8.932 to get 23.45 as the sum? — 13. _____

14. By rounding off each factor to the nearest whole number, estimate the product: — 14. _____

59.987000034 X 3.09987564

15. Write each of the following as decimal fractions: — 15. (a) _____

(a) $\frac{7}{8}$ — (b) _____

(b) $\frac{63}{72}$ — (c) _____

(c) $\frac{7}{11}$

16. By what must we multiply 0.000002 to get 0.06 as the product? — 16. _____

17. (a) How much is 40% of 80? — 17. (a) _____

(b) 40 is what percent of 80? — (b) _____

(c) 40% of what number equals 80? — (c) _____

18. What is the exact value of $23\frac{4}{7}\%$ of 14,000? — 18. _____

19. How much is 0.07% of 2,000,000? — 19. _____

20. (a) What percent of the whole is $\frac{19}{40}$? — 20. (a) _____

(b) What percent of the whole is $\frac{59}{40}$? — (b) _____

(cont)

FINAL MASTERY REVIEW: (cont)

ANSWERS:

21. At a rate of \$4.50 for $3\frac{3}{5}$ pounds of candy,

    (a) what is the price of 1 pound of candy?

    (b) what is the price of 20 pounds of candy?

    (c) how many pounds of candy can you buy for \$20?

21. (a) _____

    (b) _____

    (c) _____

22. (a) What price must you pay for a radio if the price is \$78 plus tax and the tax rate is 4%?

    (b) If the radio costs \$78 including the tax and the tax rate is 4%, what is the cost of the radio without the tax?

    (c) At 40% off the regular price, a radio costs \$78. What is the regular price of the radio?

22. (a) _____

    (b) _____

    (c) _____

23. A salesperson earns an 18% commission on all the clothes she sells.

    (a) If she sells \$450 worth of clothes, how much will her commission be?

    (b) If her commission is \$450, how many dollars worth of clothes did she sell?

23. (a) _____

    (b) _____

24. You travel at a speed of 50 miles per hour.

    (a) How far will you travel in 3 hours and 36 minutes?

    (b) How long will it take you to travel 120 miles?

24. (a) _____

    (b) _____

25. (a) How many grams are there in 1 kilogram?

    (b) There are 454 grams in 1 pound. To the nearest ounce, how many pounds are there in 24 kilograms? There are 16 ounces/pound.

    (c) To the nearest gram, how many kilograms are there in 24 pounds?

25. (a) _____

    (b) _____

    (c) _____

(cont)

FINAL MASTERY REVIEW: (concluded)

ANSWERS:

26. (a) How many square feet are there in 2,304 square inches?

(b) If there are 2.54 centimeters per inch, how many cubic centimeters are there in 4 cubic inches? Round off your answer to the nearest whole number.

(c) If the area of a square is 100 square centimeters, what is its perimeter?

26. (a) _____

(b) _____

(c) _____

27. *(Use as facts that $1.08^7 \doteq 1.7138$ and $1.08^8 \doteq 1.8509$)*

You invest $2,500 at an interest rate of 8% compounded annually.

(a) To the nearest dollar, how much is your investment worth at the end of 7 years?

(b) To the nearest dollar, how much is it worth at the end of 8 years?

(c) To the nearest dollar, by how much did your investment increase during the 8th year?

27. (a) _____

(b) _____

(c) _____

28. Let 3.1416 stand for π. If the area of a circle is 640 square feet, what is the radius of the circle? Write your answer rounded off to the nearest inch.

28. _____

29. Find the value of n if:

(a) $7(n + 5) = 77$

(b) $9n + 7 = 88$

(c) $n^2 + 5 = 41$

29. (a) _____

(b) _____

(c) _____

30. In the formula: $m = b(c + d) + e$ find the value of b if $m = 88$ when $c = 3$, $d = 6$, and $e = 7$.

30. _____

(ANSWERS ARE ON THE NEXT PAGE)

Answers for the Final Mastery Review:

1. 45,236,000,089,027

2. 5,828

3. 302

4. 156

5. (a) 2 X 2 X 3 X 3 X 3 X 5
   (b) 2 X 2 X 3 X 3 X 5 X 5
   (c) $\frac{3}{5}$

6. (a) $\frac{15}{40}$
   (b) $\frac{31}{40}$

7. (a) $\frac{1}{5}$
   (b) $\frac{2}{3}$ by $\frac{1}{5}$

8. (a) $\frac{15}{56}$
   (b) $\frac{15}{56}$
   (c) yes

9. (a) $\frac{21}{20}$
   (b) $\frac{21}{20}$
   (c) $\frac{20}{21}$

10. (a) $33\frac{3}{7}$
    (b) $\frac{228}{7}$

11. $21\frac{8}{15}$

12. (a) $\frac{12}{5}$
    (b) $\frac{12}{5}$
    (c) $\frac{51}{25}$

13. 14.518

14. 180

15. (a) 0.875
    (b) 0.875
    (c) $0.\overline{63}$ (that is, 0.636363....)

16. 30,000

17. (a) 32
    (b) 50(%)
    (c) 200

18. 3,300

19. 1,400

20. (a) 47.5% (or 47½%)
    (b) 147.5%

21. (a) $1.25
    (b) $25
    (c) 16

22. (a) $81.12
    (b) $75
    (c) $130

23. (a) $81 (b) $2,500

24. (a) 180 (b) 2.4 hrs or 2 hrs and 24 mins

25. (a) 1,000 (b) 52 lbs, 14 oz
    (c) 10 kg, 896 grams

26. (a) 16 (b) 66 (c) 40 cms

27. (a) $4,285 (b) $4,627 (c) $342

28. 14 ft 3 in (that is 14.2757 feet)

29. (a) n = 6 (b) n = 9 (c) n = 6

30. b = 9

Solutions for Final Mastery Test:

1.

In terms of our adjective-noun format, we have:

45 trillion, 236 billion, no millions, 89 thousands, and 27 units. Hence:

_ 4 5 2 3 6 _ _ _ _ 8 9 _ 2 7

trillions billions millions thousands units

As long as the denominations are visible we don't need 0's as place holders.

and if we now use commas as places holders for the denominations, we get:

45,236,000,089,027

If you wrote 45,236,089,027 you'd have 45 billion rather than 45 trillion.

2.

In terms of fill-in-the-blank, we have:

_____ + 8,389 = 14,217

which means the same as

_____ = 14,217 - 8,389

= 5,828

So while this may at first seem to be an addition problem, it is really a subtraction problem.

3.

Again in terms of fill-in-the-blank, we have:

_____ X 64 = 19,328

which means the same as:

____ = 19,328 ÷ 64

= 302

This is another form of the long division problem:

```
      3 0 2
   ---------
64)1 9 3 2 8
   1 9 2
   -------
       1 2
     - 0 0
     -----
       1 2 8
     - 1 2 8
     -------
           0
```

4.

If we divide 39,580 by 704 we get:

```
        5 6
   ---------
704)3 9 5 8 0
  - 3 5 2 0
  -------
    4 3 8 0
  - 4 2 2 4
  ---------
      1 5 6   This is the remainder.
```

If you used the calculator you got 56.221591. To find the remainder, you have to multily the decimal remainder (.221591) by 704.

5.

(a)
$$\begin{array}{r} 2)\overline{540} \\ 2)\overline{270} \\ 3)\overline{135} \\ 3)\overline{45} \\ 3)\overline{15} \\ 5 \end{array}$$

Therefore:

$540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$

You don't have to factor in the given order. For example, you could have divided by 5 first and then by 3 to obtain:

$$\begin{array}{r} 5)\overline{540} \\ 3)\overline{108} \\ 3)\overline{36} \\ 3)\overline{12} \\ 2)\overline{4} \\ 2 \end{array}$$

and so on.

(b)
$$\begin{array}{r} 2)\overline{900} \\ 2)\overline{450} \\ 3)\overline{225} \\ 3)\overline{75} \\ 5)\overline{25} \\ 5 \end{array}$$

Therefore:

$900 = 2 \times 2 \times 3 \times 3 \times 5 \times 5$

(c) We have:

$$\frac{540}{900} = \frac{\cancel{2} \times \cancel{2} \times \cancel{3} \times \cancel{3} \times 3 \times \cancel{5}}{\cancel{2} \times \cancel{2} \times \cancel{3} \times \cancel{3} \times \cancel{5} \times 5} = \frac{3}{5}$$

We're cancelling common factors in order to reduce to lowest terms.

6.

(a)

We can multiply numerator and denominator of a common fraction by the sane non-zero number without changing the value of the fraction. Since 40 is the 5th multiple of 8, we have:

$$\frac{3}{8} = \frac{3 \times 5}{8 \times 5} = \frac{15}{40}$$

(b)

The key to adding common fractions is to use common denominators. 40 is the 5th multiple of 8 and the 8th multiple of 5. Hence:

$$\frac{3}{8} = \frac{3 \times 5}{8 \times 5} = \frac{15}{40} \text{ and } \frac{2}{5} = \frac{2 \times 8}{5 \times 8} = \frac{16}{40}$$

Hence:

$$\frac{3}{8} + \frac{2}{5} = \frac{15}{40} + \frac{16}{40} = \frac{31}{40}$$

That is:

15 fortieths
+16 fortieths
31 fortieths

7.

(a)

In terms of fill-in-the-blank, we have:

$$\_\_\_\_ + \frac{7}{15} = \frac{2}{3}$$

This is the same as:

$$\begin{aligned} \_\_\_\_ &= \frac{2}{3} - \frac{7}{15} \\ &= \frac{10}{15} - \frac{7}{15} \\ &= \frac{10 - 7}{15} \\ &= \frac{3}{15} \\ &= \frac{1}{5} \end{aligned}$$

$\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}$

We have to use common denominators when we add or subtract.

We cancelled a 3 from the numerator and denominator

(b)

This is just another way of stating part (a). We showed that we had to add $\frac{1}{5}$ to $\frac{7}{15}$ to get $\frac{2}{3}$. That means that $\frac{2}{3}$ exceeds $\frac{7}{15}$ by $\frac{1}{5}$.

8.

(a)

We don't need common denominators when we multiply or divide fractions. We multiply by multiplying the numerators and multiplying the denominators. Hence:

$$\frac{3}{8} \times \frac{5}{7} = \frac{3 \times 5}{8 \times 7} = \frac{15}{56}$$

(b)

This is another form of part (a).

That is, $\frac{3}{8}$ of $\frac{5}{7}$ means $\frac{3}{8} \times \frac{5}{7}$

Remember that "of" means "X"

8 (c)

$\frac{3}{8} \times \frac{5}{7}$ means $\frac{3}{8}$ of $\frac{5}{7}$. $\frac{3}{8}$ of a number means that we take less than the number itself. In fact we're only taking 3 parts out of each 8.

A more direct way is to notice that 3/8 X 5/7 = 15/56 while 5/7 = 40/56. 15/46 is less than 40/56.

9.

(a)

We can use the invert-and-multiply rule to obtain:

$$\frac{3}{4} \div \frac{5}{7} = \frac{3}{4} \times \frac{7}{5}$$

$$= \frac{21}{20}$$

(b)

This is another form for part (a). Namely $\frac{3}{4} \div \frac{5}{7}$ means the number we must multiply by $\frac{5}{7}$ to get $\frac{3}{4}$ as the product.

Check: $\frac{5}{7} \times \frac{21}{20} = \frac{\cancel{3}^{1} \times \cancel{21}^{3}}{\cancel{7}_{1} \times \cancel{20}_{4}} = \frac{3}{4}$

If you didn't recognize this fact, use the fill-in-the-blank form to get:

$$\_\_\_ \times \frac{5}{7} = \frac{3}{4}$$

which means the same as

$$\_\_\_ = \frac{3}{4} \div \frac{5}{7}$$

The relationship between multiplication and division is the same whether we use whole numbers or fractions.

(c)

Again, using the fill-in-the-blank format:

$$\_\_\_ \times \frac{3}{4} = \frac{5}{7}$$

$$\_\_\_ = \frac{5}{7} \div \frac{3}{4}$$

$$= \frac{5}{7} \times \frac{4}{3}$$

$$= \frac{20}{21}$$

Notice the importance of keeping things in the right order. When we interchange 3/4 and 5/7 the answers are reciprocals of one another.

10.

(a)

We have to divde 234 by 7 and we get:

$$7\overline{)234}\quad 33R3 = 33\frac{3}{7}$$

$$-21$$
$$24$$
$$-21$$
$$3$$

(b)

$$32\frac{4}{7} = 32 + \frac{4}{7}$$

$$= \frac{32}{1} + \frac{4}{7}$$

$$= \frac{32 \text{ X } 7}{1 \text{ X } 7} + \frac{4 \text{ X } 1}{7 \text{ X } 1}$$

$$= \frac{224}{7} + \frac{4}{7}$$

$$= \frac{224 + 4}{7}$$

$$= \frac{228}{7}$$

Short Form:

Multiply 32 by 7
Add 4
Write the answer "over" 7

Check: $\frac{228}{7)228}\quad 32R4 = 32\frac{4}{7}$

$$- 21$$
$$18$$
$$- 14$$
$$4$$

11.

The best approach is to convert everything to common fractions, solve the resulting common fraction problem, and then convert the answer back to a mixed number. That is:

$$7\frac{3}{5} \text{ X } 2\frac{5}{6} = \frac{38}{5} \text{ X } \frac{17}{6}$$

$$= \frac{\overset{19}{\cancel{38}} \text{ X } 17}{5 \text{ X } \underset{3}{\cancel{6}}}$$

$$= \frac{323}{15)323}\quad 21R8 = 21\frac{8}{15}$$

$$-30$$
$$23$$
$$-15$$
$$8$$

You don't have to cancel common factors first. It just simplifies the computation.

Note that you can't just multiply the 2 whole numbers and the 2 fractions when you multiply mixed numbers.

12.

(a)

The decimal point holds the place of the denomination. Thus:

2.4 means 24 tenths or $\frac{24}{10}$.

If we cancel a 2 from both numerator and denominator, we get $\frac{12}{5}$.

Alternatively, 2.4 means $2\frac{4}{10}$ which also leads to 24/10.

(b)

$$2.40 = 2\frac{40}{100}$$

$$= \frac{(2 \times 100) + 40}{100}$$

$$= \frac{240}{100}$$

$$= \frac{24}{10}$$

which is the same problem as part (a).

Notice that 2.4 and 2.40 mean the same thing since the 0 doesn't change the place value of either the 2 or the 4.

(c)

$$2.04 = \frac{204}{100}$$

$$= \frac{4 \times 51}{4 \times 25}$$

$$= \frac{51}{25}$$

However when we write 2.04 the 4 is now in the hundredths place; whereas in 2.4 and 2.40 the 4 was in the tenths place.

13.

Again it makes no difference whether we use whole numbers, fractions or decimals; addition and subtraction have the same relationship:

$$\_\_\_\_\_ + 8.932 = 23.45$$

means

$$\_\_\_\_\_ = 23.45 - 8.932$$

$$= 14.518$$

If you're not using a calculator remember to write 23.45 as 23.450. That is:

$$\begin{array}{r} 23.450 \\ -\ 8.932 \\ \hline 14.518 \end{array}$$

14.

To the nearest whole number:

$59.987000034 \doteq 60$

and $3.09987564 \doteq 3$

Hence the product is approximately

60×3 or 180.

So to make the estimate, we round off first and then we multiply. The point is that if we make a mistake in counting the digits, we still know that the answer is around 180.

15.

(a)

$$\begin{array}{r} 7\ 0.875 \\ 8)\overline{7.0000} \\ -6\ 4 \\ \hline 60 \\ -\ 56 \\ \hline 40 \\ -\ 40 \\ \hline 0 \end{array}$$

We can stop here because there is no remainder.

7/8 means 7 ÷ 8 and if we use a calculator we get 0.875 at once.

(b)

Since $\frac{63}{72}$ reduces to $\frac{7}{8}$, we get the same answer here as we did in part (a).

If you didn't notice this, you can still divide to obtain:

$$\begin{array}{r} 0.875 \\ 72)\overline{63.000} \\ -57\ 6 \\ \hline 5\ 40 \\ -\ 5\ 04 \\ \hline 360 \\ -\ 360 \\ \hline 0 \end{array}$$

The key point here is that equivalent common fractions always have the same decima fraction form.

(c)

$$\begin{array}{r} 7\ 0.63\ldots \\ 11)\overline{7.00000} \\ -6\ 6 \\ \hline 40 \\ -33 \\ \hline 7 \end{array}$$

We started with a 7 followed only by 0's and we have this form again. So the cycle 63 will repeat endlessly.

Don't round off to 0.64. 0.64 is 64/100 or 16/25, which is not exactly the same as 7/11.

16.

In terms of fill-in-the-blank, we have:

____ X 0.000002 = 0.06

which means the same as

_____ = 0.06 ÷ 0.000002

= 60,000 ÷ 2

= 30,000

Move the decimal point 6 places to the right in both numbers in the ratio

That is, as small as 0.06 may seem to be, it is still 30,000 times greater than 0.000002

We can get the answer in once step without moving the decimal point if we use the calculator.

17.

The key here is to remember that "percent" tells us to divide by 100.

(a)

40% of 80 = ____

means

$\frac{40}{100}$ X 80 = ____

$\frac{3,200}{100}$ = ____; or ____ = 32

Using decimals, the problem becomes 0.40 X 80 = 32

Another approach is to notice that 1% of 80 is 0.8, so 40% of 80 is 0.8 X 40 or 32.

(b)

40 per 80 is $\frac{40}{80}$ or $\frac{1}{2}$; and $\frac{1}{2}$ of 100% = 50%

In other words, at a rate of 40 out of each 80, you'd take 50 out of each 100. But 50 per hundred means 50%

(c)

We have:

40% of _____ = 80; or

0.40 X _____ = 80; or

_____ = 80 ÷ 0.40

= 200

If 40% is 80, then 1% is 2. Hence 100% is 2 X 100 or 200.

Notice how easy it is to confuse parts (a), (b), and (c) if you don't understand what you're doing.

18.

$$23\frac{4}{7} = \frac{(23 \times 7) + 4}{7}$$

$$= \frac{161 + 4}{7}$$

$$= \frac{165}{7}$$

Hence $23\frac{4}{7}\% = 23\frac{4}{7} \div 100$

$$= \frac{165}{7} \times \frac{1}{100}$$

$$= \frac{165}{700}$$

That is $23\frac{4}{7}\%$ is just another way of saying that we're taking 165 out of each 700. Since 14,000 is the 20th multiple of 700, we're taking 165 X 20 or 3,300

Hence:

$23\frac{4}{7}\%$ of 14,000 =

$\frac{165}{700} \times 14{,}000 =$

$\frac{33}{\cancel{140}} \times \overset{100}{\cancel{14{,}000}} =$

33 X 100 or 3,300

$\frac{165}{700} = \frac{5 \times 33}{5 \times 140} = \frac{33}{140}$

19.

0.07% means

0.07 ÷ 100, or 0.0007

0.07% means 0.07 per 100, which is the same rate as 7 per 10,000

Therefore:

0.07% of 2,000,000

means

0.0007 X 2,000,000 = 1,400

See what's happening here? We're taking 7 per 10,000. 2,000,000 is the 200th multiple of 10,000. Hence, we are taking 7, 200 times or 1,400.

If you prefer to work with common fraction forms, we have: $\frac{0.07}{100} \times 2{,}000{,}000$ or

0.07 X 20,000 or

1,400

20.

The key here is that the "whole" is denoted by 100%.

(a)

So we're being asked to find: At a rate of 19 out of each 40, how many will we take out of each 100?

19 per 40 =
38 per 80 =
57 per 120

So we know that out of 100 we take more than 38 but less than 57.

This leads to the arithmetic computation:

$\frac{19}{40}$ of 100% =

$$\begin{array}{r} 1,900 \quad 4\ 7.5\% \\ 40)1\ 9\ 0\ 0.0 \\ -1\ 6\ 0 \\ \hline 3\ 0\ 0 \\ -\ 2\ 8\ 0 \\ \hline 2\ 0\ 0 \\ -\ 2\ 0\ 0 \\ \hline 0 \end{array}$$

You could write this as 47½%

(b)

$\frac{59}{40}$ of 100% =

$$\begin{array}{r} 5,900 \quad 1\ 4\ 7.5 \\ 40)5\ 9\ 0\ 0.0 \\ -\ 4\ 0 \\ \hline 1\ 9\ 0 \\ -\ 1\ 6\ 0 \\ \hline 3\ 0\ 0 \\ -\ 2\ 8\ 0 \\ \hline 2\ 0\ 0 \\ -\ 2\ 0\ 0 \\ \hline 0 \end{array}$$

In other words if your return is \$59 per \$40 you invested, you'd get \$147.50 for every \$100 you invested

21.

(a)

"dollars per pound" tells us to divide dollars by pounds. That is:

4.50 dollars ÷ $3\frac{3}{5}$ pounds =

(4.50 ÷ $3\frac{3}{5}$) dollars per pound -

21 (a) (cont)

$\frac{450}{100} \div \frac{18}{5}$ dollars per pound =

$\frac{450}{100} \times \frac{5}{18}$ dollars per pound =

$\frac{450 \times 5}{100 \times 18}$ dollars per pound =

$\frac{2,250}{1,800}$ $\frac{1.25}{2\,2\,5\,0.\,0\,0}$ dollars per pound

$-\ 1\,8\,0\,0$

$4\,5\,0\,0$

$-\ 3\,6\,0\,0$

$9\,0\,0\,0$

$-\ 9\,0\,0\,0$

0

In terms of decimals $3\frac{3}{5} = 3.6$ Hence we can find the answer at once using the calculator by computing $4.50 \div 3.6$

We could cancel common factors first but I preferred to use the calculator.

For parts (b) and (c) we can use the formula:

# of pounds X $1.25 = total cost in dollars.

(b)

We replace the # of pounds in the formula by 20 to get:

20 X $1.25 = $25

As a quick check you know that at more than $1 per pound, it has to cost more than $20 for 20 pounds.

(c)

Now we replace the total cost by $25 to get:

____ X $1.25 = $25, or:

_____ = $25 ÷ $1.25

= 20 (pounds)

It is easy to confuse parts (a) and (b) if you don't read carefully.

22.

You're paying $1.04 for each dollar the object is marked. Hence the formula--for either part (a) or part (b)--is:

1.04 X Marked Price = Total Price

22. (cont)

(a)

Here we replace the Marked price by $78 to get:

1.04 X $78 = total price; or

$81.12 = total price

(b)

This time we replace the Total price by $78 to get:

1.04 X Marked Price = $78; or

Marked price = $78 ÷ 1.04

= $75

You can do these problems longhand but it's much quicker to use the calculator.

(c)

40% off the regular price means that we only pay $0.60 for each $1 of the regular price. That is:

$0.60 X Regular Price = Sale Price

Since the $78 is the sale price, we replace the Sale Price by 78 in our formula to get:

$0.60 X Regular Price = $78

Regular Price = $78 ÷ $0.60

= $130

For many people this is the hardest part of the problem--trying to translate the given information into a formula.

23.

For both parts of this problem, we shall use the formula:

18% of Total Sales = The Commission

23. (cont)

Or in the language of decimals:

0.18 X Total Sales = The Commission

(a)

Here we're told that the Total Sales is $450. Hence the formula becomes:

0.18 X $450 = The Commission

$81 = The Commission.

Another way to think of this of this problem is that she makes 18¢ per $1 of sales. That is, her commission is $18 per $100 or $72 per $400 and $90 per $500. This tells us that per $450 she makes less than $90 but more than $72.

(b)

This time it's the commission that's $450. So now the formula becomes:

0.18 X Total Sales = $450

Total Sales = $450 ÷ 0.18

= $2,500

That is, she makes $18 per $100 of sales. Since $450 is the 25th multiply of $18, the total sales are the 25th multiple of $100 or $2,500.

24.

We want to use either both miles and hours or both miles and minutes. We do not want to mix minutes and hours. If we use hours the formula will be

50 mph X # of hours = # of miles

In doing part (a) we will convert 3 hours and 36 minutes into 3 hours plus 36 sixtieths of an hour or $3\frac{36}{60}$ or 3.6 hours. This is a convenient form if we're using a calculator but we could write the time as $3\frac{3}{5}$ hours and use mixed numbers. Hence:

If you prefer to use minutes, 50 mph is 50/60 or 5/6 miles per minute; hence:

$\frac{5}{6}$ *miles/min X # of minutes = # of miles*

If we use minutes, we'd write the time as (60 X 3) plus 36 or 216 minutes.

24. (a)

We replace the number of hours by 3.6 in our formula to get:

50 mph X 3.6 hours =

(50 X 3.6) miles = 180 miles

As a rough check you'd go 150 miles in 3 hours and 200 miles in 4 hours, so in 3 hours and 36 minutes you'd go between 150 and 200 miles. So 180 miles is at least reasonable.

That is:

$\frac{miles}{hr}$ *X hrs = miles*

(b)

This time we replace the number of miles by 120 and our formula becomes:

50 mph X # of hours = 120 miles

# of hours = 120 ÷ 50

= 2.4 hours

or in terms of minutes, 2 hours and 24 minutes.

Rough estimate of the answer: It takes 2 hours to go 100 miles and 3 hours to go 150 miles. So it takes between 2 and 3 hours to go 120 miles.

0.4 hours = 0.4 X 60 minutes or 24 minutes.

25.

(a)

The definition of "kilo" is "1,000. Hence a kilogram is 1,000 grams.

(b)

In terms of filling in the blank we have:

24 kgs = ___ lbs.

Recalling that common labels can cancel, we have:

25 (b) (cont)

24 kgs =

$$\frac{24 \text{ kgs}}{1} \times \frac{1{,}000 \text{ gm}}{1 \text{ kg}} \times \frac{1 \text{ lb}}{454 \text{ gm}} =$$

$$\frac{24 \times 1{,}000}{454} \text{ pounds} =$$

$$\frac{24{,}000}{454} \text{ pounds} =$$

52.863436 pound =

52 pounds + 0.863436 pounds =

52 pounds + 0.863436 X 16 ounces =

52 pounds and 14 ounces

More briefly:

Start with 24 (kg)
Multiply by 1,000 (gms)
Divide by 454 (pounds)

Roughly speaking 0.9 pounds would be 0.9 X 16 ounces or 14 ounces. Do not confuse 16.9 pounds with 16 pounds and 9 ounces.

(c)

This time we start with 24 pounds; then we multiply by 454 to get grams; and then we divide by 1,000 to get kilograms; That is:

$$\frac{24 \text{ pounds}}{1} \times \frac{454 \text{ grams}}{1 \text{ pound}} \times \frac{1 \text{ kilogram}}{1{,}000 \text{ grams}} =$$

$$\frac{24 \times 454}{1{,}000} \text{ kilograms} =$$

$$\frac{10{,}896}{1{,}000} \text{ or } 10.896 \text{ kilograms}$$

and since there are 1,000 grams in a kllogram, the answer can be written as 10 kgs and 896 grams

26.

(a)

A square foot means 1 foot X 1 foot or 12 inches X 12 inches. Thus there are 144 square inches per square yard.

So we divide 2,304 square inches by 144 to get 16 square feet as our answer.

$$\frac{2{,}304 \text{ sq in}}{1} \times \frac{1 \text{ sq ft}}{144 \text{ sq in}} =$$

$$\frac{2{,}304}{144} \text{ sq ft}$$

26. (b)

1 cubic inch = 1 inch X 1 inch X 1 inch

= 2.54 cm X 2.54 cm X 2.54 cm

= 2.54^3 cubic centimeters (cc)

= 16.387064 cc

4 cubic inches = 4 X 16.387064 cc

= 65.548256 cc

≐ 66 cc

(c)

Since $10^2 = 100$, if the area of a square is 100 square centimeters, the length of each side of the square must be 10 centimeters. Since the perimeter of a square is 4 times the length of any side, the perimeter will be 4 X 10 centimeters or 40 centimeters .

27.

The key in this problem is that after n years the value of your investment will be

1.08^n X \$2,500.

(a)

Using 7 for n we have:

1.08^7 X \$2,500 = 1.7138 X \$2,500

≐ \$4,285

27. (b)

This time we replace n by 8 to get:

$1.08^8 \times \$2,500 = 1.8509 \times \$2,500$

$\doteq \$4,627$

(c)

To find what you earned during the 8th year you take what you had at the end of the 8th year ($4,627) and subtract what you had at the beginning of the 8th year ($4,285). We get:

$\$4,627 - \$4,285 = \$342.$

The beginning of the 8th year is the same as the end of the 7th year.

28.

To find the area of a circle we square the radius, and then multiply by π. Reversing the steps, if we know the area, we divide by π first and then take the square root. In this case:

Start with 640 square feet

Divide by 3.1416 (203.71785 sq ft)

Take the square root (14.272976 feet)

So the radius is $\doteq$ 14.273 feet, and since we want the anwer to the nearest inch, we rewrite 0.273 feet as 0.273 X 12 inches or about 3 inches.

29.

(a)

Since the 7 is multiplying the parentheses, we first divide both sides by 7 to get $n + 5 = 11$. We then subtract 5 from both sides to get $n = 6$.

To undo adding 5 we subtract 5 from both sides of the equation.

29 (b)

This means (9n) + 7 = 88. So we first "unblock" the parentheses by subtracting 7 from both sides to get:

9n = 81

and we then undo multiplying n by 9, by dividing both sides by 9; and we get n = 9.

See the steps?

Start with n
Multiply by 9
Add 7
The answer is 88

To undo this:

Start with 88
Subtract 7
Divide by 9
The answer is n.

(c)

Given $(n^2) + 5 = 41$, we first subtract 5 from both sides to get $n^2 = 36$; and we then take the square root of both sides to get that n = 6.

The "program"is:

Start with n
Square it
Add 5
The answer is 41

To undo this "program"

Start with 41
Subtract 5
Take the square root
The answer is n

30.

This simply shows us how we get from a formula to an equation. In this case we take the formula m = b(c + d) + e and replace each variable by its given value. We get:

88 = b(3 + 6) + 7

or

88 = 9b + 7

or

9b + 7 = 88

This is precisely the same problem as part (b) of the previous problem, except that the name of the variable has been changed from n to b. So the answer will be, b = 9.

It is not important what name we give to the variable (except that in a given problem we don't want the same letter to name two different variables)

FINAL MASTERY REVIEW REVISITED:

ANSWERS:

1. Write twenty four trillion, eight hundred thirty seven million, fifty six thousand, sixty nine as a place value numeral.

1. _____

2. What number must we add to 7,978 to get 16,234 as the sum?

2. _____

3. What number must we multiply by 76 to 38,228 as the product?

3. _____

4. What is the remainder when 38,400 is divided by 704?

4. _____

5. (a) Write 1,575 as a product of prime numbers.

   (b) Write 1,050 as a product of prime numbers.

   (c) Write $\frac{1,050}{1,575}$ as a common fraction in lowest terms.

5. (a) _____

   (b) _____

   (c) _____

6. (a) Write $\frac{4}{7}$ as an equivalent common fraction whose denominator is 63.

   (b) Write $\frac{4}{7} + \frac{2}{9}$ as a common fraction.

6. (a) _____

   (b) _____

7. (a) What must we add to $\frac{4}{9}$ to get $\frac{2}{3}$?

   (b) Which is greater and by how much: $\frac{2}{3}$ or $\frac{4}{9}$?

7. (a) _____

   (b) _____

8. (a) Write $\frac{4}{7} \times \frac{2}{9}$ as a common fraction.

   (b) How much is $\frac{4}{7}$ of $\frac{2}{9}$?

   (c) Is $\frac{4}{7} \times \frac{2}{9}$ more than $\frac{2}{9}$?

8. (a) _____

   (b) _____

   (c) _____

9. (a) Write $\frac{3}{7} \div \frac{4}{5}$ as a common fraction.

   (b) By what must we multiply $\frac{4}{5}$ to get $\frac{3}{7}$?

   (c) By what must we multiply $\frac{3}{7}$ to get $\frac{4}{5}$?

9. (a) _____

   (b) _____

   (c) _____

10. (a) Write $\frac{423}{11}$ as a mixed number.

    (b) Write $46\frac{3}{4}$ as a common fraction.

10. (a) _____

    (b) _____

(cont)

FINAL MASTERY REVIEW REVISITED (cont):

11. Write $8\frac{2}{3} \times 5\frac{4}{5}$ as a mixed number.

12. Write each of the following as a common fraction in lowest terms:
    (a) 3.6
    (b) 3.06
    (c) 3.60

13. What must we add to 19.234 to get 37.112 as the sum?

14. By rounding off each factor to the nearest whole number, estimate the product:
    70.09999876 X 3.9987003123

15. Write each of the following as decimal fractions:
    (a) $\frac{3}{8}$
    (b) $\frac{21}{56}$
    (c) $\frac{3}{11}$

16. By what must we multiply 0.000012 to get 0.0048 as the product?

17. (a) How much is 30% of 90?
    (b) 30 is what percent of 90?
    (c) 30% of what number equals 90?

18. What is the exact value of $23\frac{2}{3}\%$ of 90,000?

19. How much is 0.03% of 800,000?

20. (a) What percent of the whole is $\frac{17}{60}$?
    (b) What percent of the whole is $\frac{77}{60}$?

21. At a rate of \$5.60 for $4\frac{2}{3}$ pounds of candy,
    (a) how much does 1 pound of candy cost?
    (b) what is the price of 30 pounds of candy?
    (c) how many pounds of candy can you buy for \$30?

(cont)

ANSWERS:

11. _____

12. (a) _____
    (b) _____
    (c) _____

13. _____

14. _____

15. (a) _____
    (b) _____
    (c) _____

16. _____

17. (a) _____
    (b) _____
    (c) _____

18. _____

19. _____

20. (a) _____
    (b) _____

21. (a) _____
    (b) _____
    (c) _____

FINAL MASTERY REVIEW REVISITED (cont):

ANSWERS:

22. (a) What price must you pay for a radio if the price is $81 plus tax and the tax rate is 8%?

(b) If the radio costs $81 including the tax and the tax rate is 8%, what is the cost of the radio without the tax?

(c) At 46% off the regular price, a radio costs $81. What is the regular price of the radio?

22. (a) _____

(b) _____

(c) _____

23. A salesperson earns a 16% commission on all the clothes she sells.

(a) If she sells $560 worth of clothes, how much will her commission be?

(b) If her commission is $560, how many dollars worth of clothes did she sell?

23. (a) _____

(b) _____

24. You travel at a speed of 45 miles per hour.

(a) How far will you travel in 4 hours and 24 minutes?

(b) How long will it take you to travel 144 miles?

24. (a) _____

(b) _____

25. (a) How many grams are there in 2 kilograms?

(b) There are 454 grams in 1 pound. To the nearest ounce, how many pounds are there in 32 kilograms?

(c) To the nearest gram, how many kilograms are there in 32 pounds?

25. (a) _____

(b) _____

(c) _____

26. (a) How many square feet are there in 3,456 square inches?

(b) There are 2.54 centimeters in 1 inch. To the nearest whole number, how many cubic centimeters are there in 5 cubic inches?

(c) If the area of a square is 36 square feet, what is the perimeter of the square?

26. (a) _____

(b) _____

(c) _____

(cont)

ANSWERS:

27. Use the facts that $1.07^6 \doteq 1.50073$ and $1.07^7 \doteq 1.60578$.

You invest \$3,500 at an interest rate of 7% compounded annually.

(a) To the nearest dollar, how much is your investment worth at the end of 6 years?

(b) To the nearest dollar, how much is your investment worth at the end of 7 years?

(c) To the nearest dollar, by how much did your investment increase during the 7th year?

27. (a) _____

(b) _____

(c) _____

28. Let 3.1416 stand for π. If the area of a circle is 830 square feet, what is the radius of the circle? Round off your answer to the nearest inch.

28. _____

29. Find the value of n if:

(a) $8(n + 3) = 96$

(b) $6n + 3 = 57$

(c) $n^2 + 11 = 155$

29. (a) _____

(b) _____

(c) _____

30. In the formula: $m = b(c + d) + e$, find the value of b if $m = 77$ when $c = 6$, $d = 2$, and $e = 5$.

30. _____

(ANSWERS ARE ON THE NEXT PAGE)

Answers for the Final Mastery Review Revisited:

1. 24,000,837,056,069
2. 8.256
3. 503
4. 384
5. (a) 3 X 3 X 5 X 5 X 7 (b) 2 X 3 X 5 X 5 X 7 (c) $\frac{2}{3}$
6. (a) $\frac{36}{63}$ (b) $\frac{50}{63}$
7. (a) $\frac{2}{9}$ (b) $\frac{2}{3}$ by $\frac{2}{9}$
8. (a) $\frac{8}{63}$ (b) $\frac{8}{63}$ (c) no
9. (a) $\frac{15}{28}$ (b) $\frac{15}{28}$ (c) $\frac{28}{15}$
10. (a) $38\frac{5}{11}$ (b) $\frac{187}{4}$
11. $50\frac{4}{15}$
12. (a) $\frac{18}{5}$ (b) $\frac{153}{50}$ (c) $\frac{18}{5}$
13. 17.878
14. 280
15. (a) 0.375 (b) 0.375 (c) $0.\overline{27}$
16. 400
17. (a) 27 (b) $33\frac{1}{3}$(%) (c) 300
18. 21,300
19. 240
20. (a) $28.\overline{3}$% or $28\frac{1}{3}$% (b) $128.\overline{3}$%
21. (a) $1.20 (b) $36 (c) 25
22. (a) $87.48 (b) $75 (c) $150
23. (a) $89.60 (b) $3,500
24. (a) 198 miles (b) 3.2 hours or 3 hours and 12 minutes
25. (a) 2,000 (b) 70 pounds, 8 oz (c) 14 kg, 528 grams
26. (a) 24 (b) 82 (c) 24 feet
27. (a) $5,253 (b) $5,620 (c) $367
28. 16 feet 3 inches (16.254125 feet)
29. (a) n = 9 (b) n = 9 (c) n = 12
30. n = 9

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*** THE END ***

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